Bayesian Model Calibration of Cavity Flow Noise

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Data Source

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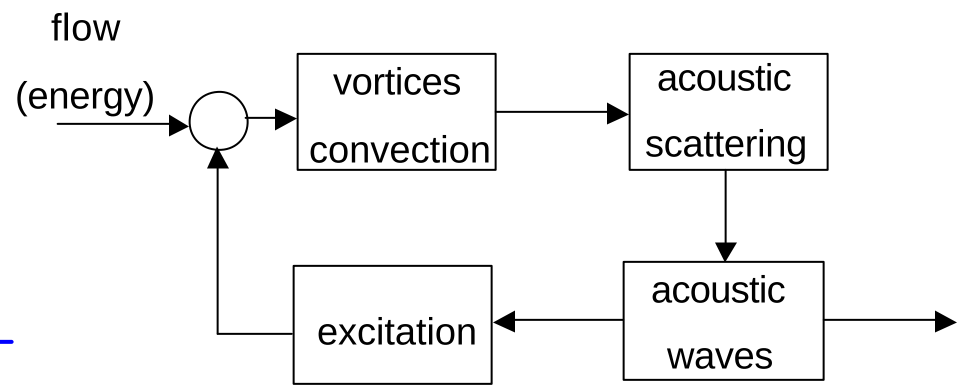
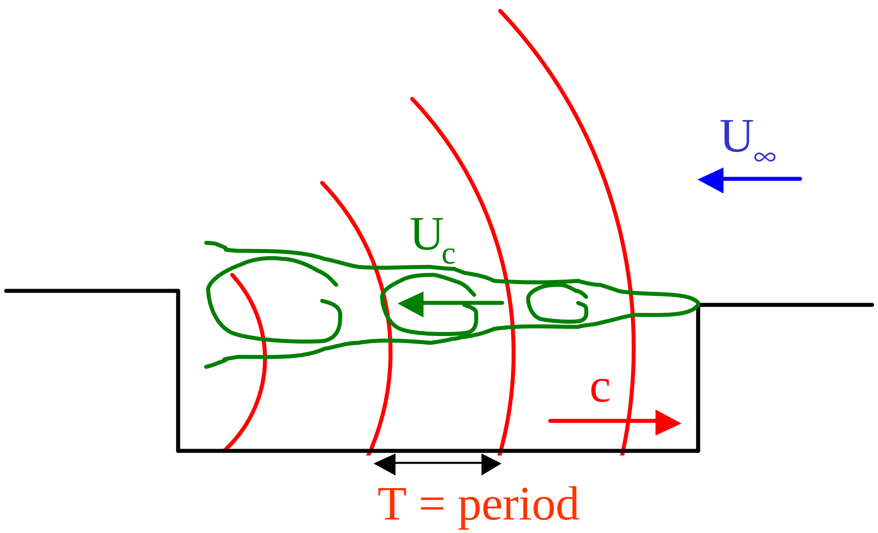
“Closed-loop, Active Flow Control (2004)”

SELF EXCITED RESONANT FLOW NOISE

Driven by feedback mechanism

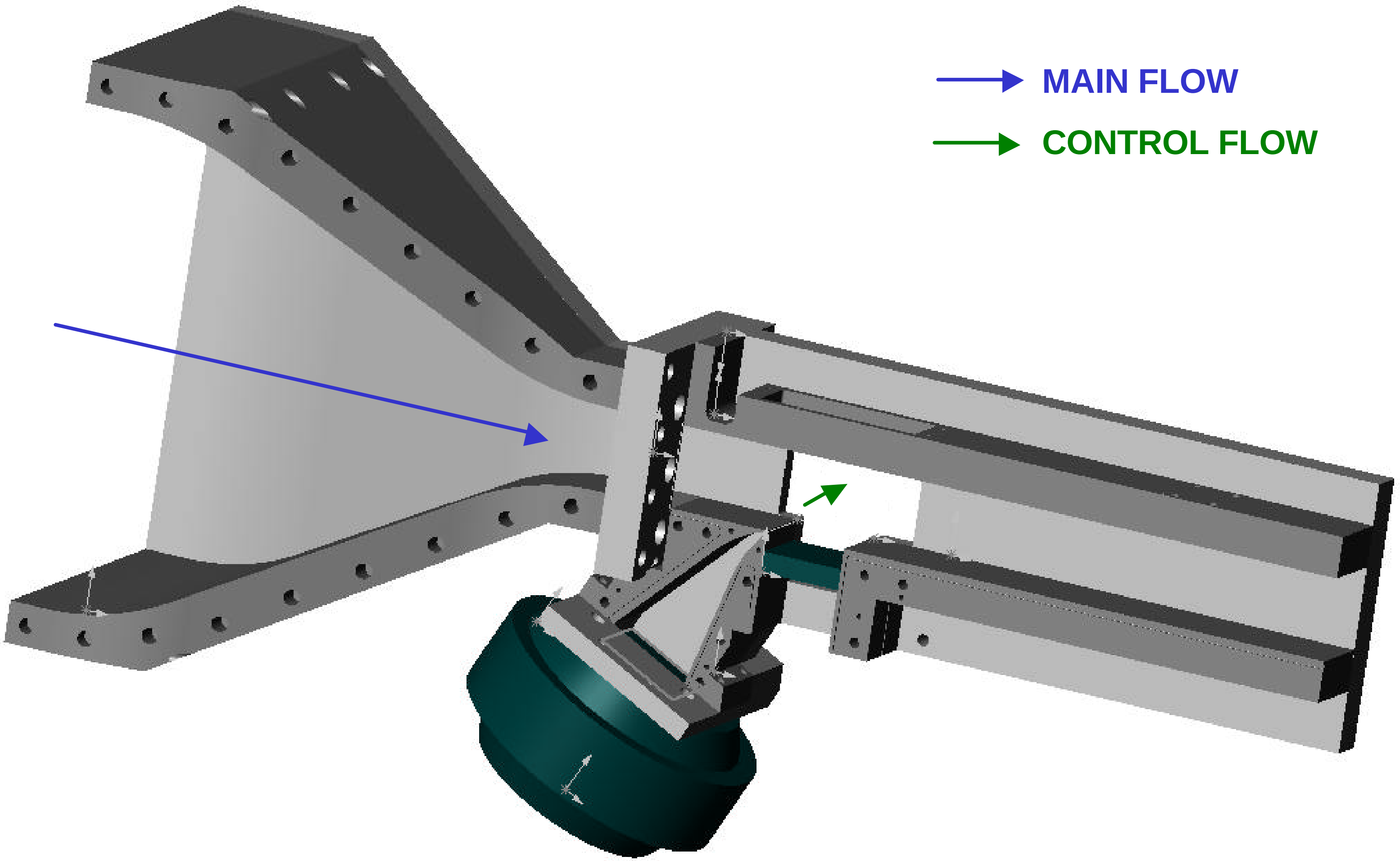




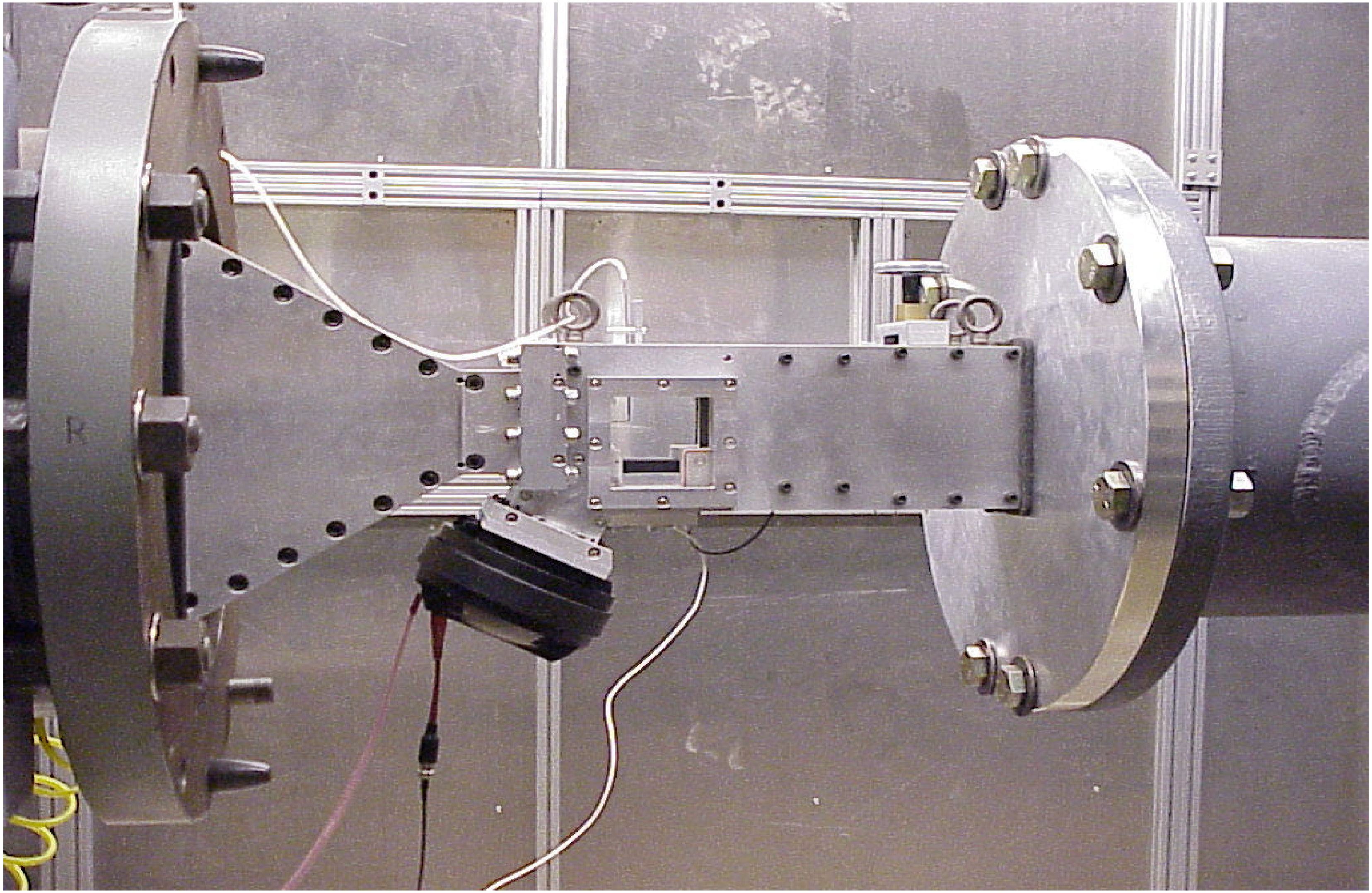
$f = 1/T \quad [1/\text{second} \equiv \text{Hz}]$

$M_\infty = U_\infty / c$

3-D VIEW OF THE TEST SECTION WITH NOZZLE AND ACTUATOR



LATERAL VIEW OF THE ASSEMBLED FACILITY



THE ROSSITER MODEL (1964)

a semi-empirical formula for resonant Strouhal number as a function of free stream Mach number

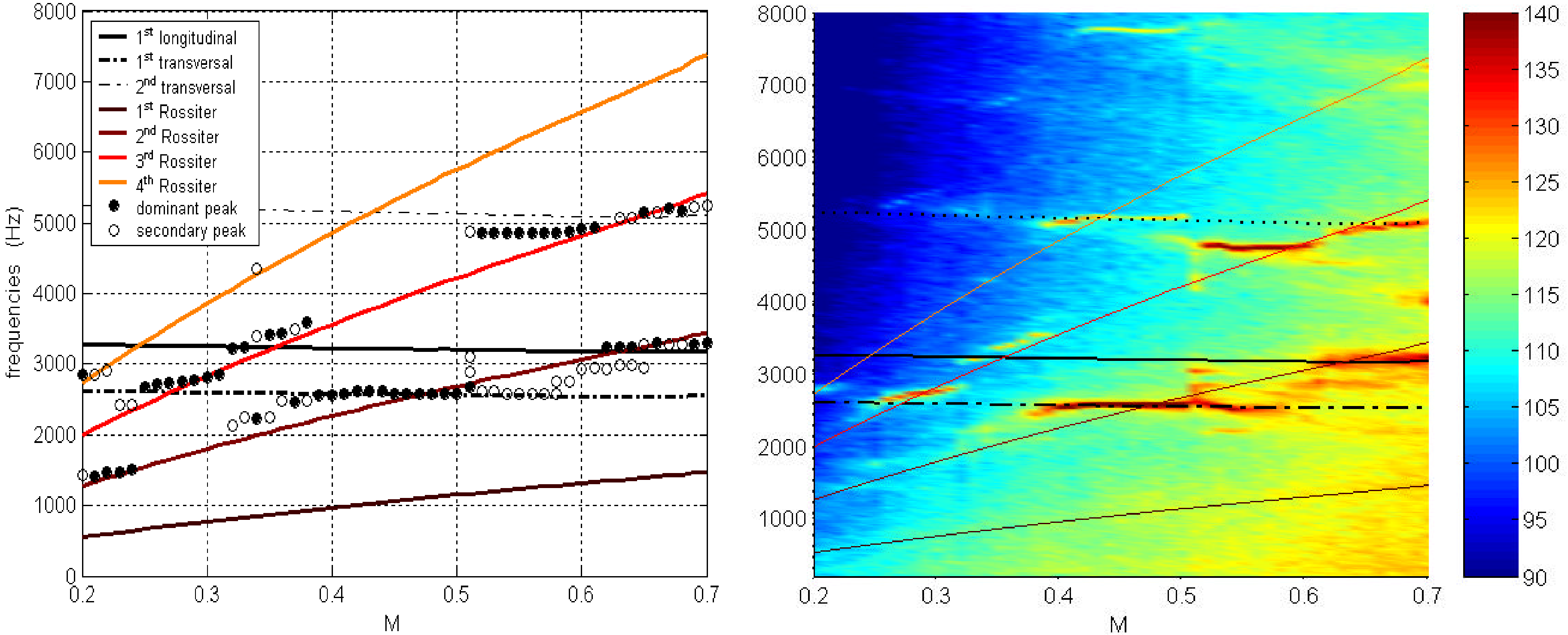
$$St_n = \frac{f_n L}{U_\infty} = \frac{n - a}{M_\infty \left(1 + \frac{g - 1}{2} M_\infty^2\right)^{-1/2} + \frac{1}{\text{ß}}}$$

u **is the mode number**

a is the **phase shift** (in fractions of a wavelength) of the acoustic scattering process

b is the **ratio** of the convective speed of the disturbance to the free stream velocity: $\beta = U_c / U_\infty$

ROSSITER’S PREDICTION AND EXPERIMENTAL RESULTS



Notation

- $\boldsymbol{X} \rightarrow$ control variable (Mach number)

  where $X \in [0.20\ , 0.70]$

  $\boldsymbol{N} \rightarrow$ mode number $\boldsymbol{N} \in \{\ 2, 3, 4\ \}$
- $\boldsymbol{Y}(\boldsymbol{x}, \boldsymbol{n}) \rightarrow$ observed freq at $\boldsymbol{X} = \boldsymbol{x}$, and $N=\boldsymbol{n}$
- $\eta(\boldsymbol{x}\ ,\boldsymbol{n}\ ,(a,b\ )) \rightarrow$ freq accd'g to Rossiter model

  where $Y(\boldsymbol{x}), \eta(\boldsymbol{x}, u\ ,(a,b)) > \mathbf{0}$
- $\mathrm{V}(\boldsymbol{x},\boldsymbol{n}) \rightarrow$ true unknown freq at $\boldsymbol{X} = \boldsymbol{x}$ and $\boldsymbol{N} = \boldsymbol{n}$

Statistical Formulation 1

- ***Y*(x_i) =** *True Freq* V(x_i , n_i) +

  Measurement error e(x_i , n_i) , ***i*=1,2,..,*n***

- ***True Freq*** V(x_i , n_i) =

  Rossiter model h(x_i , n_i, (*a*,*b*)) +

  Deviation of Rossiter model from the true process d(x_i , n_i)

Statistical formulation 2

- $Y(x_i, n_i) = \mathrm{V}(x_i, n_i) + e(x_i, n_i)$, $i=1,2,..,n$

  $e(x_i, n_i) \sim \mathrm{MVN}\ (\mathbf{0}, s^2\mathbf{I})$

  $s^2 \sim \mathrm{IG}\ (\mathbf{SA}, \mathbf{SB})$

- $\mathrm{V}(x_i, n_i) = \mathbf{B}\ \eta(x_i, n_i, (a, b)) + \delta(x_i, n_i)$

  $a \sim \mathrm{Beta}\ [\mathbf{AA}, \mathbf{AB}]$

  $b \sim \mathrm{Beta}[\ \mathbf{BA}, \mathbf{BB}]$

Statistical formulation 2

$$\delta(x_i, n_i) \sim \mathbf{MVN}(m, S)$$

$$m = \mathbf{0}$$

$$S = \{ s_{ij} \} = l^{-1} \exp(-f \, | \boldsymbol{x_i} - \boldsymbol{x_j} |^2) + k \bullet \mathbf{d}_{ij}$$

where $d_{ij} = I(i=j)$

$$l \sim \mathbf{Gamma}(LA, LB)$$

$$f \sim \mathbf{Gamma}(PA, PB)$$

$$k \sim \mathbf{Gamma}(KA, KB)$$

Parameter Vector

$$q = [\ (a\ ,\ b)\ ,\ s^2,\ l\ ,\ f, k\ ]$$

Posterior Distribution

$$\mathbf{L}\ (\ q\ |\ \boldsymbol{y}\ )\ \mu\ \mathbf{L}\ (q\ |\ \boldsymbol{y}\ ) \bullet \mathbf{[}\ q\ \mathbf{]}$$

$$\mu\ \mathbf{L}\ (q\ |\ \boldsymbol{y}\ )\ \bullet$$

$$\mathbf{[}a\mathbf{]}\ \mathbf{[}b\mathbf{]}\ \mathbf{[}s^2\mathbf{]}\ \mathbf{[}l\ \mathbf{]}\ \mathbf{[}\ f\mathbf{][}k\mathbf{]}$$

Metropolis Hastings within Gibbs

We need to sample from

i. $[\ h(\boldsymbol{x})\ |\ rest, \boldsymbol{Y}\ ] \sim \text{MVN}(\ m^*, S^*\ )$

ii. $[\ s^2\ |\ rest, \boldsymbol{Y}\ ] \sim$ Inverse Gamma

iii. $[\ (a,b)\ |\ rest, \boldsymbol{Y}\ ] \propto \exp\{-\, g_1(\ h(x_i),(a,b), (l, j, k)\ )\}$

- Metropolis-Hastings Step

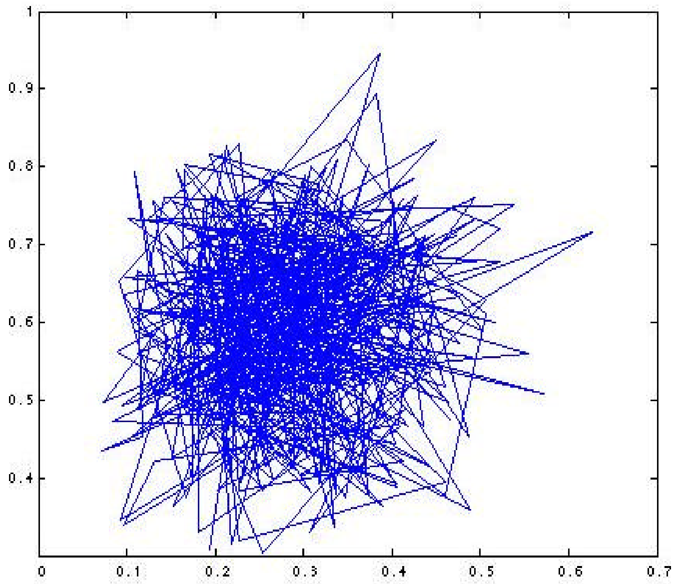
iv. $[\ (l, j, k)\ |\ rest, \boldsymbol{Y}\ ] \propto \exp\{-\, g_2(\ h(x_i),(a,b), (l, j, k)\ )\}$

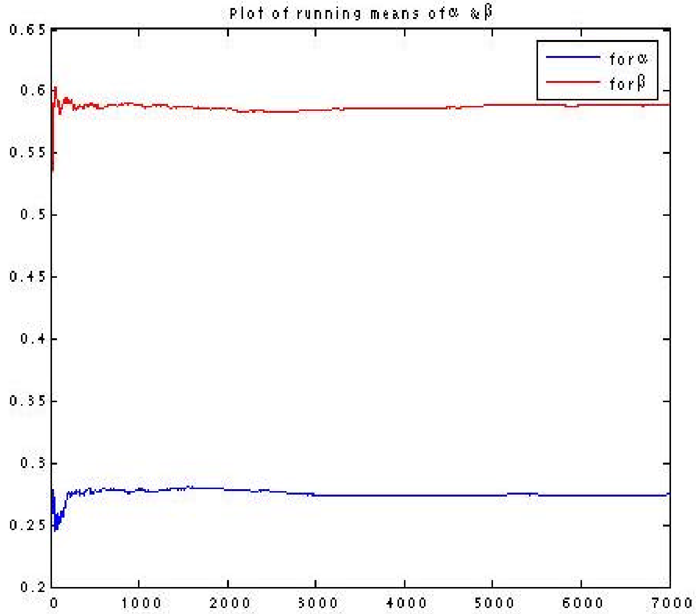
- Metropolis-Hastings Step

Priors and Initial Values

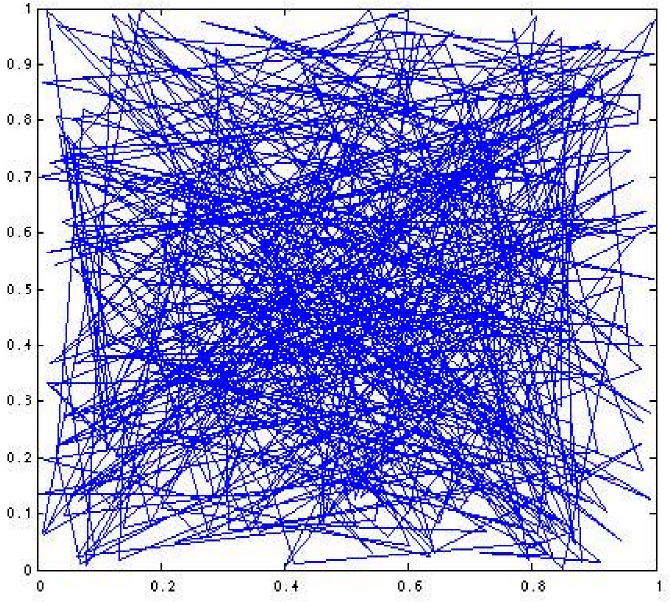
Parameter	Distribution	Hyperparameters
a	Beta (AA, AB)	Prior Mode = 0.250, 0.800 AA+AB = 10
b	Beta (BA, BB)	Prior Mode = 0.625, 0.100 BA+BB = 10
s^2	IGamma (SA, SB)	SA = 4.0 SB = 0.5
l	Gamma (LA, LB)	LA = 0.2 LB = 0.2
f	Gamma (PA, PB)	PA = 20.0 PB = 2.0
k	Gamma (KA, KB)	KA = 2.0 KB = 1.0

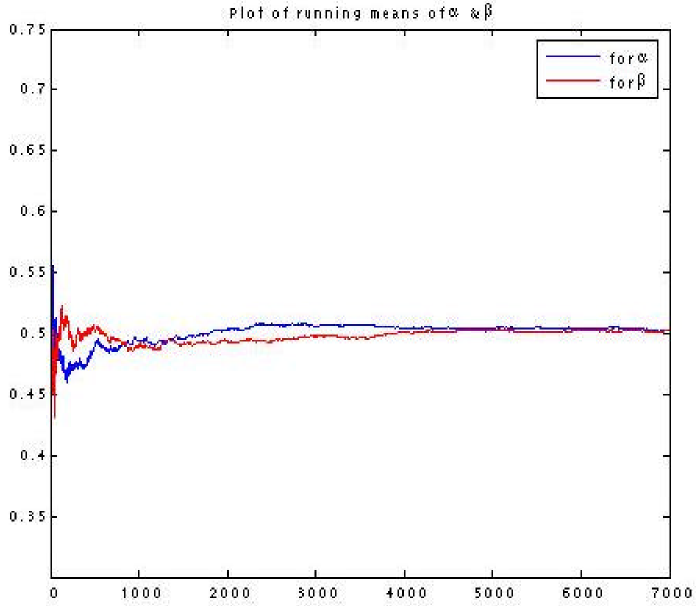
Mixing & Convergence Checks 1
Independent Beta Priors





Mixing & Convergence Checks 2 Independent Uniform Priors





Posterior Inference (Iterations =7000)

Burn-in = 2000 , ***Thin*** = 10, ***Samples*** = 500

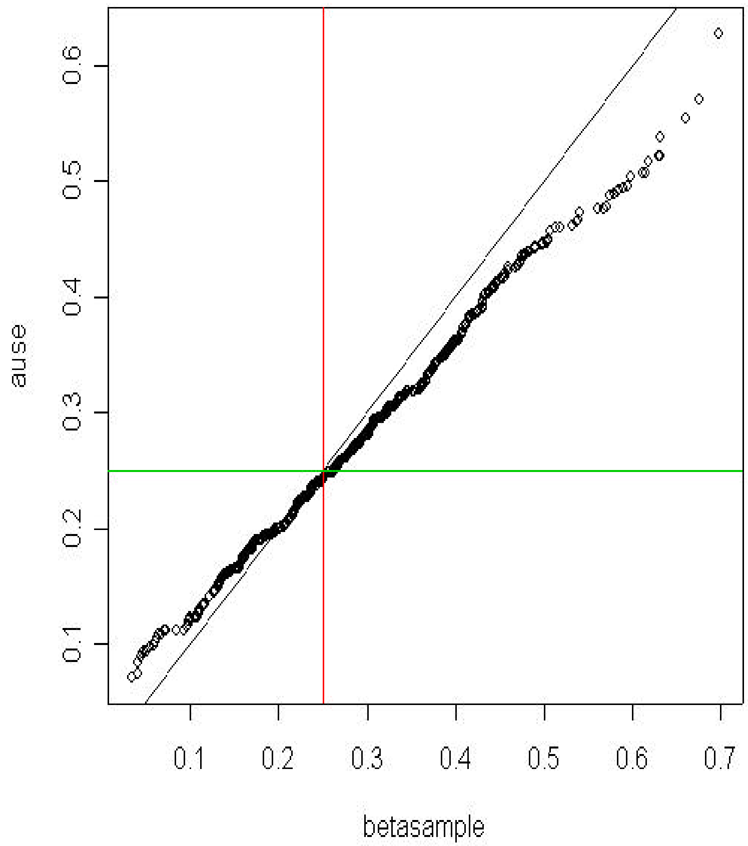
Parameter	***Mean***	***SD***	***Q1***	***Median***	***Q3***	***95% Credible Intervals***	
a	0.27	0.10	0.20	0.27	0.34	0.10	0.49
b	0.59	0.11	0.52	0.59	0.66	0.37	0.80
Note: {(AA+AB) = ***10*** *; (BA+BB) =* ***10****} {(AA+AB) =* ***10*** *; (BA+BB) =* ***10****}*							
a	0.48	0.30	0.22	0.47	0.75	0.02	0.97
b	0.51	0.30	0.26	0.52	0.76	0.03	0.98
Note: {(AA+AB) = 2 ; (BA+BB) = 2} {(AA+AB) = 2 ; (BA+BB) = 2}							

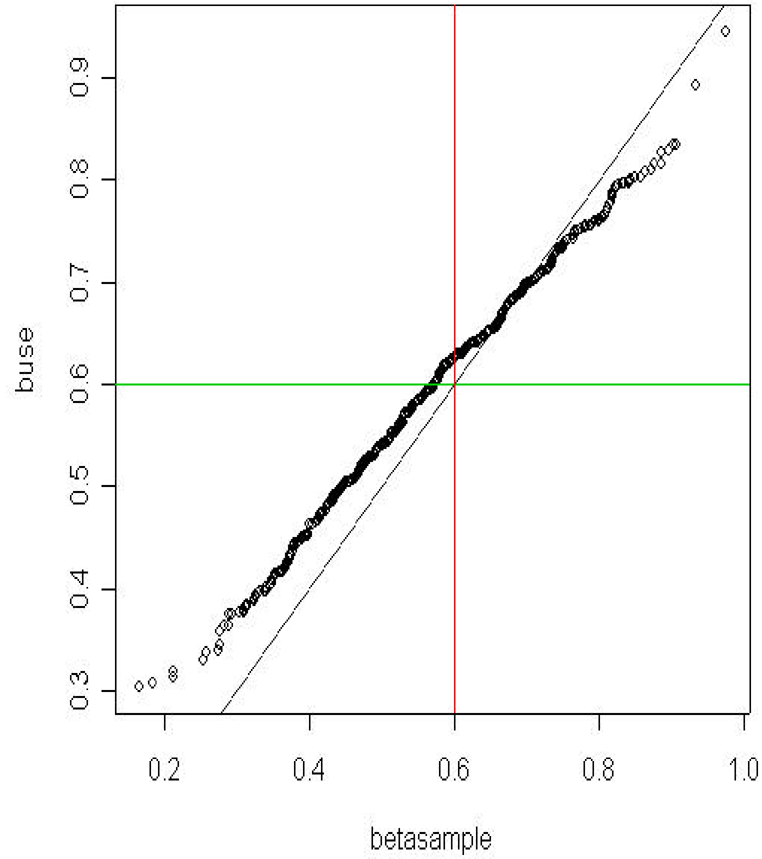
Posterior Inference (Iterations =7000)

Burn-in = 2000 , ***Thin*** = 10, ***Samples*** = 500

Parameter	*Mean*	*SD*	*Q1*	*Median*	*Q3*	***95% Credible Intervals***	
a	0.26	.06	0.22	0.26	0.30	0.15	0.38
b	0.60	0.06	0.55	0.60	0.64	0.48	0.72
*Note: {(AA+AB) = **30** ; (BA+BB) = **30**} {(AA+AB) = **30** ; (BA+BB) = **30**}* ***Mode set at** a **= 0.25 ,** b **=0.60***							
a	0.79	0.05	0.76	0.79	0.82	0.69	0.88
b	0.11	0.04	0.08	0.11	0.14	0.05	0.20
*Note: {(AA+AB) = **30** ; (BA+BB) = **30**} {(AA+AB) = **30** ; (BA+BB) = **30**}* ***Mode set at** a **= 0.80 ,** b **=0.10***							

QQ Plot of Posterior and Prior CDF of [a ,b, | y]





Summary of Findings

- *Posterior means*: *a =0.27 , b =0.59*

  MC Std Error: a =0.10 , b =0.11

  95% Credible intervals:

  a Î (0.10,0.49) ; b Î (0.37,0.80)

- Overall convergence was achieved at 2000th iteration; Quality of mixing was acceptable; Choice of thinning interval was OK
- Posterior distribution is not invariant to the choice of prior mode.

Next Steps

- Use statistical approach to choose the “peak” frequencies
- Use discriminant analysis to identify mode number in conjunction with Bayesian set-up
- Re-parameterization of space to speed up computation time, and convergence
- Try multiplicative error model for $V(\boldsymbol{x},\boldsymbol{n})$